

# Measurement-induced manipulation of the quantum-classical border

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We demonstrate the possibility of controlling the border between the quantum and the classical world by performing nonselective measurements on quantum systems. We consider a quantum harmonic oscillator initially prepared in a Schrödinger cat state and interacting with its environment. We show that the environment induced decoherence transforming the cat state into a statistical mixture can be strongly inhibited by means of appropriate sequences of measurements.

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## I. INTRODUCTION

The increasing ability in the coherent control and manipulation of the state of quantum systems has paved the way to experiments able to monitor the transition from quantum superpositions, such as the paradigmatic Schrödinger cat states, to classical statistical mixtures [1]. The emergence of the classical world from the quantum world, due to decoherence induced by the environment, has been extensively investigated in the last few decades both in connection to fundamental issues of quantum theory and in relation to the emerging quantum technologies. The fragile nature of quantum superpositions and entangled states exploited, e.g., in quantum communication, quantum computation, and quantum metrology, makes these potentially very powerful techniques also very delicate. For this reason several methods have been proposed in order to protect quantum states from decoherence and dissipation. For example, methods based on decoherence free subspaces, dynamical decoupling and bang-bang techniques, just to mention a few, have been investigated [2]. Recently the connection between these techniques and the quantum Zeno effect has been clarified [3].

In a recent letter we have studied the conditions for observing the Zeno and anti-Zeno effects in a damped harmonic oscillator [4]. The quantum Zeno and anti-Zeno effects [5, 6] predict the inhibition and the enhancement, respectively, of the decay of the initial state due to a series of measurements aimed at checking whether the system is still in its initial state or not [7]. Typically, when studying Zeno and anti-Zeno dynamics, the system is assumed to be initially prepared in an eigenstate of the free Hamiltonian, e.g., in our case, a Fock state. This is the situation we have considered in Refs. [4, 8]. The aim of this paper is to see whether the quantum Zeno effect can be exploited also to inhibit quantum decoherence when the system is initially prepared in a Schrödinger cat state. The analysis of Zeno and anti-Zeno phenomena in the context of the damped harmonic oscillator gives us the possibility of exploring the modification of the quantum-classical transition as an effect of measurements performed on the system. This possibility stems from the fact that the harmonic oscillator possesses both

quantum states, such as Fock states and superposition of coherent states, and classical (or semiclassical) states, such as the coherent and the thermal states.

Another aspect discussed in the paper is the connection between the dynamics in presence of non-selective energy measurements and the dynamics in presence of modulation of the system-reservoir coupling constant. The second scenario may be useful in implementing experiments aimed at revealing the quantum Zeno and anti-Zeno effects with engineered reservoirs [1]. We will show that a simple periodic modulation of the system-reservoir coupling constant is equivalent to performing non-selective energy measurements, as one would expect from the results presented in Ref. [3]. An experimental verification of the Zeno and anti-Zeno effects with engineered reservoirs would allow one to observe these phenomena through indirect measurements, contrarily to the direct ones of Ref. [9], in the spirit of the ‘genuine’ quantum Zeno effect [10].

## II. QUANTUM ZENO AND ANTI-ZENO EFFECTS FOR THE DAMPED HARMONIC OSCILLATOR

We consider a harmonic oscillator linearly coupled with a reservoir modelled as an infinite chain of non-interacting oscillators [11, 12, 13, 14, 15, 16, 17]. The dynamics of the damped harmonic oscillator is described, in the secular approximation and in the interaction picture, by means of the following generalized master equation [18, 19]

$$\begin{aligned} \frac{d\rho(t)}{dt} = & \frac{\Delta(t)+\gamma(t)}{2} [2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a] \\ & + \frac{\Delta(t)-\gamma(t)}{2} [2a^\dagger \rho(t)a - aa^\dagger \rho(t) - \rho(t)aa^\dagger]. \end{aligned} \quad (1)$$

In this equation,  $a$  and  $a^\dagger$  are the annihilation and creation operators, and  $\rho(t)$  is the reduced density matrix of the system oscillator. It is worth noticing that the only approximation done in the derivation of the master equation (1) is the secular approximation typical of quantum optical systems. However, no Born-Markov approxima-

tion has been done, therefore this master equation describes the non-Markovian dynamics of the system. For times much longer than the reservoir correlation time  $\tau_R$ , the time dependent coefficients  $\Delta(t) + \gamma(t)$  and  $\Delta(t) - \gamma(t)$  approach their stationary values

$$\gamma_1^M \equiv \Gamma[N(\omega_0) + 1], \quad (2)$$

$$\gamma_{-1}^M \equiv \Gamma N(\omega_0), \quad (3)$$

respectively, [See Appendix A of Ref. [19] for details] and the master equation (1) reduces to the well known Markovian master equation for the damped harmonic oscillator.

In contrast to other non-Markovian dynamical systems, the master equation (1) is local in time, i.e. it does not contain memory integrals. All the non-Markovian character of the system is contained in the time dependent coefficients appearing in the master equation. These coefficients, namely the diffusion coefficient  $\Delta(t)$  and the damping coefficient  $\gamma(t)$ , depend uniquely on the form of the reservoir spectral density. To second order in the dimensionless system-reservoir coupling constant  $g$ , they take the form [17, 19]

$$\Delta(t) = g^2 \int_0^t \int_0^\infty d\omega dt_1 J(\omega) [2N(\omega) + 1] \cos(\omega t_1) \cos(\omega_0 t_1), \quad (4)$$

$$\gamma(t) = g^2 \int_0^t \int_0^\infty d\omega dt_1 J(\omega) \sin(\omega t_1) \sin(\omega_0 t_1), \quad (5)$$

with  $J(\omega)$  the reservoir spectral density,  $N(\omega) = (e^{\hbar\omega/k_B T} - 1)^{-1}$  the average number of reservoir thermal photons,  $k_B$  the Boltzmann constant, and  $T$  the reservoir temperature. We note that, for high  $T$ , i.e.  $N(\omega) \gg 1$ ,  $\Delta(t) \gg \gamma(t)$ .

### A. Evolution in presence of non-selective energy measurements

The time evolution of this system has been extensively studied [13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24], and in particular the dynamics of initial nonclassical states due to environment induced decoherence has been discussed in various regimes. Here we describe the modifications of the open system dynamics due to a series of non-selective energy measurements, described in terms of the projection operator  $\hat{P}$

$$\hat{P}\rho = \sum_n P_n |n\rangle\langle n|, \quad (6)$$

where  $|n\rangle$  are the Fock states of the harmonic oscillator and  $P_n = \langle n|\rho|n\rangle$  are the diagonal elements of the reduced density matrix. Essentially the effect of these measurements is to erase instantaneously all the coherences, without selecting any of the energy states of the systems. We assume that during the time evolution the harmonic oscillator is subjected to a series of non-selective energy measurements, and we indicate with  $\tau$  the time interval between two successive measurements. We assume that

$\tau$  is so short (and/or the coupling so weak) that second order processes may be neglected.

Following the derivation given, for a generic system, in Ref. [3] we can write down a coarse grained master equation governing the system time evolution in presence of  $m$  nonselective measurements

$$\begin{aligned} \frac{d\rho(t)}{dt} = & \gamma_1(\tau) \hat{P} \left[ a \hat{P} \rho(t) a^\dagger - \frac{1}{2} a^\dagger a \hat{P} \rho(t) - \frac{1}{2} \hat{P} \rho(t) a^\dagger a \right] \\ & + \gamma_{-1}(\tau) \hat{P} \left[ a^\dagger \hat{P} \rho(t) a - \frac{1}{2} a a^\dagger \hat{P} \rho(t) - \frac{1}{2} \hat{P} \rho(t) a a^\dagger \right]. \end{aligned} \quad (7)$$

Here  $t = m\tau$  and  $\gamma_{\pm 1}(\tau)$  are given by

$$\gamma_{\pm 1}(\tau) = \tau \int_{-\infty}^{\infty} d\omega \kappa^\beta(\omega) \text{sinc}^2 \left( \frac{\omega \mp \omega_0}{2} \tau \right), \quad (8)$$

where  $\text{sinc}(x) = \sin x/x$ , and the thermal spectral density  $\kappa^\beta(\omega)$  is defined as

$$\kappa^\beta(\omega) = J(\omega) \theta(\omega) [N(\omega) + 1] + J(-\omega) \theta(-\omega) N(-\omega), \quad (9)$$

with  $\theta(\omega)$  the unit step function.

The dynamics described by the master equation (7) is such that only the diagonal elements of the density matrix are nonzero, due to the effect of the nonselective measurement described by Eq. (6). The time evolution of the number probability distribution  $P_n(t) = \langle n|\rho(t)|n\rangle$  is easily obtained by Eq. (7), and reads as

$$\begin{aligned} \dot{P}_n(t) = & \gamma_1(\tau) [(n+1)P_{n+1}(t) - nP_n(t)] \\ & + \gamma_{-1}(\tau) [nP_{n-1}(t) - (n+1)P_n(t)]. \end{aligned} \quad (10)$$

We note that the decay rates  $\gamma_1(\tau)$  and  $\gamma_{-1}(\tau)$  do not depend on time  $t$ , hence these rate equations are formally equivalent to those obtained from the Markovian master equation for the damped harmonic oscillator, provided that one identifies  $\gamma_1(\tau)$  with  $\Gamma[N(\omega_0) + 1]$  and  $\gamma_{-1}(\tau)$  with  $\Gamma N(\omega)$ . The effect of the nonselective energy measurements is therefore twofold. On the one hand they destroy the off diagonal elements of the density matrix, and on the other hand they modify the decay coefficients appearing in the rate equations in a way which depends crucially both on the system/reservoir parameters and on the interval  $\tau$  between the measurements. In order to understand in more detail how the decay coefficients are modified when compared to the Markovian ones we further investigate  $\gamma_{\pm 1}(\tau)$ , as given by Eq. (8) with Eq. (9). Recasting Eqs. (4)-(5) as

$$\begin{aligned} \Delta(t) = & \frac{t}{2} \int_0^\infty d\omega J(\omega) \left[ N(\omega) + \frac{1}{2} \right] \left\{ \text{sinc}[(\omega - \omega_0)t] \right. \\ & \left. + \text{sinc}[(\omega + \omega_0)t] \right\}, \end{aligned} \quad (11)$$

$$\gamma(t) = \frac{t}{2} \int_0^\infty d\omega \frac{J(\omega)}{2} \left\{ \text{sinc}[(\omega - \omega_0)t] - \text{sinc}[(\omega + \omega_0)t] \right\} \quad (12)$$

and integrating the sum and the difference of these coefficients over the time interval  $\tau$ , it is straightforward to prove that

$$\gamma_{\pm 1}(\tau) = \frac{1}{\tau} \int_0^\tau dt [\Delta(t) \pm \gamma(t)]. \quad (13)$$

The equation above shows the connection between the coefficients of the non-Markovian master equation (1) and the coefficients  $\gamma_{\pm}(\tau)$  modified by the presence of the nonselective energy measurements. We note first of all that when the interval between the measurements  $\tau$  is much greater than the reservoir correlation time  $\tau_R$ ,  $\gamma_{\pm 1}(\tau) \simeq \gamma_{\pm 1}^M$ , since  $\Delta(t)$  and  $\gamma(t)$  quickly set to their Markovian stationary values. In this case one recovers for  $P_n(t)$  the usual Markovian dynamics, i.e., the presence of the nonselective energy measurements cannot modify the Markovian decay of the number probability distribution. Stated another way, in order to affect the dynamics one needs to perform the measurements at time intervals shorter or of the same order of the reservoir correlation time. This is well known in the theory of the quantum Zeno effect, since such effect is crucially related to the short time initial quadratic behavior of the survival probability, i.e. of the probability that a system prepared in a given initial state is still in that initial state after a time  $t$  [7].

As we mentioned at the beginning of this section for high  $T$  reservoirs  $\Delta(t) \gg \gamma(t)$ , therefore, for times much smaller than the thermalization time  $\tau_{th} \simeq 1/\Gamma$ , with  $\Gamma$  defined in Eqs. (2)-(3),

$$\gamma_1(\tau) \simeq \gamma_{-1}(\tau) \simeq \frac{1}{\tau} \int_0^\tau dt \Delta(t). \quad (14)$$

As we have noted and discussed in Ref. [4], for high  $T$  reservoirs, therefore, the modified decay rates depend only on the diffusion coefficient  $\Delta(t)$  describing environment induced decoherence. In other words the repetition of nonselective energy measurements at short intervals  $\tau$  forces the system to experience an enhanced or reduced environment induced decoherence, depending on the form of the reservoir spectrum, and hence on the temporal behavior of  $\Delta(t)$ .

Before discussing the effect of measurements on the decoherence of an initial Schrödinger cat state, we want to address the connection between the dynamics described in this section and the situation in which the measurements are replaced by a periodic modulation in the system-reservoir coupling constant.

### III. MIMICKING THE EFFECT OF MEASUREMENTS WITH ENGINEERED RESERVOIRS

We consider here the case in which the coupling between the system oscillator and the reservoir is weak enough to justify the use of second order perturbation

theory. Instead of introducing a series of nonselective energy measurements we consider the case in which the microscopic system reservoir coupling constant  $g$  is modulated as follows

$$g(t) = \begin{cases} g & \text{for } m(\tau + \delta\tau) \leq t < m(\tau + \delta\tau) + \tau, \\ 0 & \text{for } m(\tau + \delta\tau) + \tau \leq t < (m+1)(\tau + \delta\tau), \end{cases} \quad (15)$$

with  $m = 0, 1, 2, \dots$ . Stated another way, the coupling between the system and the reservoir is interrupted, for a short time  $\delta\tau \ll \tau$ , periodically at intervals  $\tau$ . As we will show in the following, under certain conditions, these short interruptions mimic the nonselective energy measurements. We refer to the system described here with the name of shuttered reservoir, according to the terminology we have used in Ref. [8]. We will assume in the following that the time intervals  $\delta\tau$  are so small that the free evolution of the system can be neglected. This assumption is not necessary when we deal with the dynamics of an initial Fock state, as we have explained in Ref. [8].

#### A. The recursive solution

Since both  $\Delta(t)$  and  $\gamma(t)$  are proportional to the coupling constant  $g$  [See Eqs. (4)-(5)], we can solve the dynamics using recursively the solution of the master equation Eq. (1). More in detail, we solve the master equation given by Eq. (1), e.g. in terms of the quantum characteristic function (QCF) as done in Ref. [18], and we use the solution at time  $\tau$  as initial condition at time  $\tau + \delta\tau$ . We then calculate the solution at time  $2\tau + \delta\tau$  and use this as initial condition once more. In this way we obtain the following coarse grained recursive solution for the QCF,

$$\chi_{m,\tau}(\xi) = \exp[-f_m(\tau)|\xi|^2] \chi_0(e^{-m\Gamma(\tau)\xi/2}), \quad (16)$$

where  $m$  is the number of interruptions in the system-reservoir coupling,  $\chi_0(\xi)$  is the QCF of the initial state, and

$$f_m(\tau) = \Delta_\Gamma(\tau) \frac{1 - e^{-m\Gamma(\tau)}}{1 - e^{-\Gamma(\tau)}}, \quad (17)$$

with

$$\Gamma(\tau) = 2 \int_0^\tau dt \gamma(t), \quad (18)$$

$$\Delta_\Gamma(\tau) = e^{-\Gamma(\tau)} \int_0^\tau dt e^{\Gamma(t)} \Delta(t). \quad (19)$$

#### B. Comparison between the shuttered reservoir and the nonselective measurements scenarios

It is straightforward to demonstrate that the density matrix corresponding to the QCF solution given by

Eq. (16) is the solution of the following master equation

$$\begin{aligned} \frac{d\rho(t)}{dt} = & \gamma_1(\tau) \left[ a\rho(t)a^\dagger - \frac{1}{2}a^\dagger a\rho(t) - \frac{1}{2}\rho(t)a^\dagger a \right] \\ & + \gamma_{-1}(\tau) \left[ a^\dagger \rho(t)a - \frac{1}{2}aa^\dagger \rho(t) - \frac{1}{2}\rho(t)aa^\dagger \right]. \end{aligned} \quad (20)$$

In order to prove it one only needs to transform the master equation above into a partial differential equation for the QCF [See, e.g., Appendix 12 of Ref. [25]], and then to verify by direct substitution that Eq. (16) satisfies the partial differential equation.

Comparing Eq. (20), describing the dynamics in presence of a shuttered reservoir, with Eq. (7), describing the dynamics in presence of nonselective energy measurements, one notices immediately that the difference between the two physical situations consists in the fact that while the nonselective measurements always set to zero the coherences, in the case of a shuttered reservoir the off diagonal elements of the density matrix do not vanish. The rate equations for the number probability distribution  $P_n(t)$ , however, do coincide, and are given in both cases by Eq. (10). This leads us to two conclusions.

Firstly, whenever the initial state of the system is a Fock state, or any state which is diagonal in the Fock state basis, the system dynamics in presence of shuttered reservoirs coincides exactly with the dynamics in presence of nonselective energy measurements, since for this type of initial condition the density matrix, for the shuttered reservoir case, remains diagonal at all times  $t$ . Therefore, in this case, the shuttered reservoir mimics the Zeno/anti-Zeno dynamics in presence of nonselective energy measurements. This is interesting because it may be easier to realize experimentally a shuttered reservoir, instead of a sequence of nonselective energy measurements, by using reservoir engineering techniques as those used in the trapped ions context [1]. In Refs. [4, 8] we briefly discuss a possible implementation of the shuttered reservoir with trapped ions.

Secondly, the time evolution of the number probability distribution, and therefore of all the observables which are diagonal in the Fock state basis, coincides for the two scenarios discussed in this paper. In Sec. IV B we will examine in detail the dynamics of the number probability distribution for the case of an initial Schrödinger cat state, and we will show that for certain values of the parameters one can manipulate the quantum-classical border prolonging or shortening the ‘life’ of the cat.

#### IV. CONTROL OF THE QUANTUM-CLASSICAL BORDER

In this section we consider the case in which the harmonic oscillator is initially prepared in a Schrödinger cat

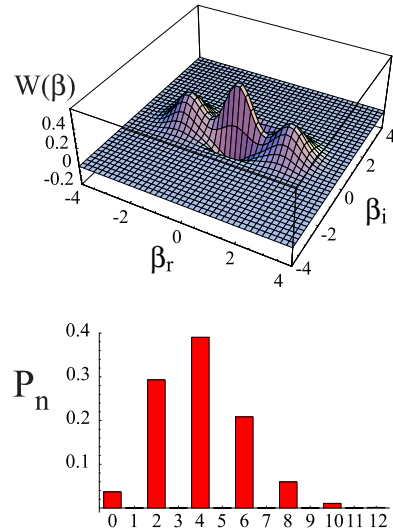


FIG. 1: (Color on line) Wigner function  $W(\beta)$  and number probability distribution  $P_n$  for the state given by Eq. (21) with  $\alpha = 2$ .

state of the form

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}} (|\alpha\rangle + |-\alpha\rangle), \quad (21)$$

where  $|\alpha\rangle$ , with  $\alpha \in \mathbb{R}$  is a coherent state and

$$\mathcal{N}^{-1} = 2 [1 + \exp(-2|\alpha|^2)]. \quad (22)$$

This state is also known as even coherent state due to the fact that only the even components of the number probability distribution are nonzero. These oscillations in the number state probability are a strong sign of the nonclassicality of this state. This and other nonclassical properties of the even coherent state, such as the negativity of the corresponding Wigner function, have been extensively studied in the literature [See, e.g., Ref. [26] and references therein]. In particular, the decoherence and dissipation due to the interaction with the environment, in the Markovian approximation, have been studied in Ref. [26], and the non-Markovian dynamics has been discussed in Refs. [19, 20]. In Fig. 1 we show the number probability distribution and the Wigner function for the even coherent state. This state has been realized in the trapped ion context and the transition from a quantum superposition to a classical statistical mixture has been observed experimentally [1].

#### A. Zeno and anti-Zeno dynamics of Schrödinger cat states: the Wigner function

We now focus on the shuttered reservoir scenario, since we believe that the realization of an experiment in this context might be already within the grasp of the experimentalists. The time evolution of the Wigner function can be calculated in two equivalent ways. One can either

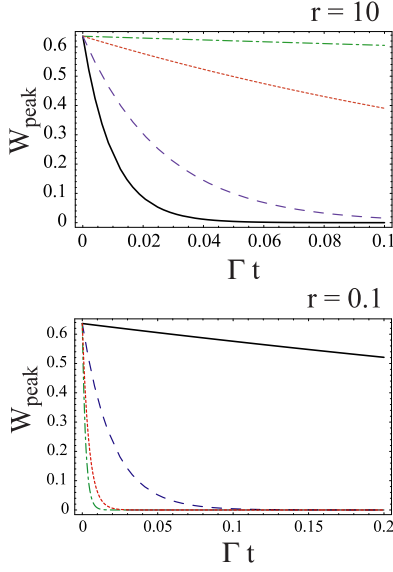


FIG. 2: (Color online) Dynamics of the Wigner function peak for a high  $T$  Ohmic reservoir, and for an initial even coherent state with  $\alpha = 2$ , in correspondence of the parameters  $r = 10$  (figure above) and  $r = 0.1$  (figure below). In both cases the solid (black) line represents the Markovian dynamics in absence of shuttering. The dashed (blue) line corresponds to  $\omega_c \tau = 1$ , the dotted (red) line corresponds to  $\omega_c \tau = 0.1$  and the dash-dotted (green) line corresponds to  $\omega_c \tau = 0.01$ .

use the QCF recursive solution given by Eq. (16), since the Wigner function is the Fourier transform of the QCF,

$$W(\beta) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} d^2 \xi \chi(\xi) \exp(\beta \xi^* - \beta^* \xi). \quad (23)$$

Or, alternatively, one can use the Markovian solution for the Wigner function corresponding to the master equation (20) [See, e.g., Ref. [26]], and replace the Markovian decay coefficients with  $\gamma_{\pm 1}(\tau)$ . The analytical expression for the Wigner function at time  $t$ , for a system initially prepared in the state given by Eq. (21) and subjected to the shuttered reservoir, is the following

$$W(\beta, t) = W^{(\alpha)}(\beta, t) + W^{(-\alpha)}(\beta, t) + W_I(\beta, t), \quad (24)$$

with

$$W^{\pm \alpha}(\beta, t) = \frac{2\mathcal{N}}{\pi(2a_t + 1)} \exp\left(-\frac{2\beta_i^2}{2a_t + 1}\right) \times \exp\left[-\frac{2(\beta_r \mp e^{-b_\tau t/2}\alpha)^2}{2a_t + 1}\right], \quad (25)$$

$$W_I(\beta, t) = \frac{2\mathcal{N}}{\pi(2a_t + 1)} \exp\left(-\frac{2|\beta|^2}{2a_t + 1}\right) \times \exp\left[-2\left(1 - \frac{e^{-b_\tau t}}{2a_t + 1}\right)\alpha^2\right] \times \cos\left(\frac{4e^{-b_\tau t/2}}{2a_t + 1}\alpha\beta_i\right). \quad (26)$$

In the previous equations  $\beta_r$  and  $\beta_i$  are the real and imaginary part of  $\beta$ , respectively, and the functions  $a_t$  and  $b_\tau$  are defined as

$$a_t = \frac{\gamma_{-1}(\tau)}{b_\tau} [1 - e^{-b_\tau t}], \quad (27)$$

$$b_\tau = \gamma_1(\tau) - \gamma_{-1}(\tau). \quad (28)$$

We now look at the decay of the interference peak of the Wigner function at  $\beta = (0, 0)$ , defined as

$$W_{\text{peak}}(t) = W_I(\beta, t)|_{\beta=(0,0)}. \quad (29)$$

For a high  $T$  reservoir,  $W_{\text{peak}}(t)$ , during the initial moments of the evolution, can be approximated as follows

$$W_{\text{peak}}(t) \simeq \frac{4\mathcal{N}}{\pi} \exp[-2\gamma_{-1}(\tau)(1 + 2\alpha^2)t]. \quad (30)$$

We consider a reservoir with an Ohmic spectral density with Lorentzian cutoff [11]

$$J(\omega) = \frac{2\omega}{\pi} \frac{\omega_c^2}{\omega_c^2 + \omega^2}, \quad (31)$$

with  $\omega_c$  the cutoff frequency. We note, incidentally, that the reservoir correlation time  $\tau_R$  is given, in this case, by the inverse of the cutoff frequency. Under these conditions we obtain the following analytic expression

$$\gamma_{-1}(\tau) = \frac{\Gamma N(\omega_0)}{\tau \omega_c} \left\{ \omega_c \tau + \frac{1 - r^2}{1 + r^2} [1 - e^{-\omega_c \tau} \cos(\omega_0 \tau)] - \frac{2r}{1 + r^2} e^{-\omega_c \tau} \sin(\omega_0 \tau) \right\}, \quad (32)$$

where

$$\Gamma = 2g^2 \frac{r^2}{r^2 + 1} \omega_0, \quad (33)$$

$\omega_0$  is the frequency of the system oscillator, and  $r = \omega_c/\omega_0$ . As we have noted already in Refs. [4, 8], the occurrence of Zeno or anti-Zeno effects strongly depends on the value of the parameter  $r$ . Values of  $r$  smaller than unity indicate that the frequency of the system oscillator is “detuned” from the spectrum of the reservoir, while values of  $r$  greater than unity indicate an overlap between the reservoir frequency spectrum and  $\omega_0$ . Inserting Eq. (32) into Eq. (30) one obtains the analytic expression for the time evolution of the peak of the Wigner function for the shuttered reservoir case.

We notice that, when the shuttering period  $\tau$  is much longer than the reservoir correlation time, i.e.  $\omega_c \tau \gg 1$ , one recovers the Markovian expression for the Wigner function peak dynamics in absence of shuttering. Stated another way, as we have already mentioned in this paper, for  $\omega_c \tau \gg 1$  the shuttering does not affect the dynamics. On the other hand, for values of  $\tau$  such that  $\omega_c \tau \leq 1$ , one observes a change in the Wigner peak dynamics depending on the behavior of the coefficient  $\gamma_{-1}(\tau)$ , given by Eq. (32).

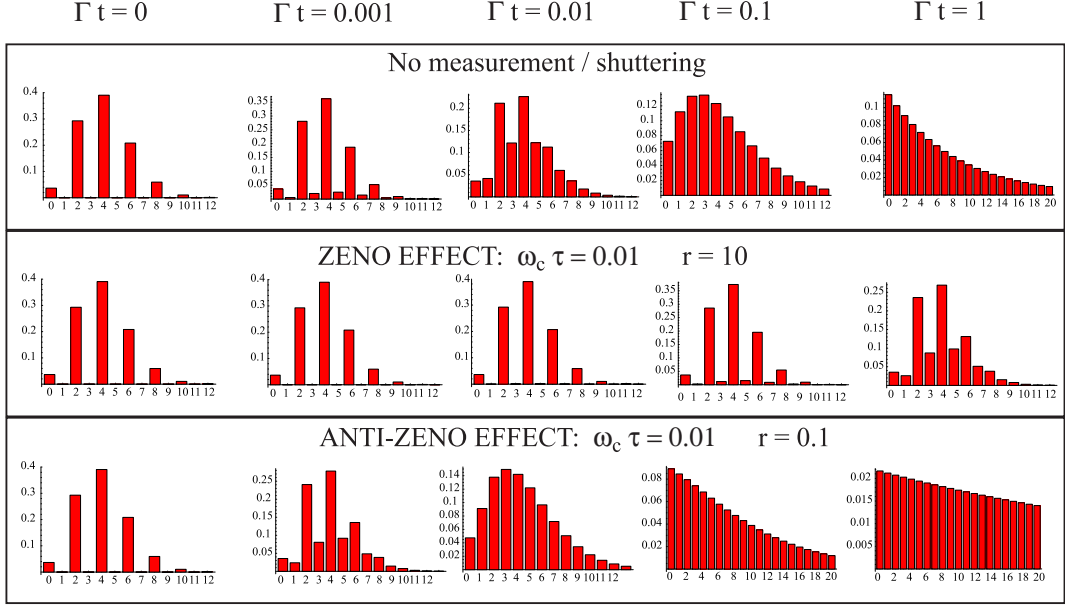


FIG. 3: (Color online) Number probability distributions at different times  $t$  for the case of no measurements/shuttering (upper row), for the case of measurements/shuttering with period  $\omega_c \tau = 0.01$  and  $r = 10$  (middle row) or  $r = 0.1$  (lower row), respectively.

In Fig. 2 we compare the Markovian dynamics in absence of shuttering and in presence of shuttering for  $\omega_c \tau = 0.1$ . For  $r = 10$ , i.e., in the case of a “resonant” reservoir the decay of the peak of the Wigner function, indicating the passage from a quantum superposition to a statistical mixture, is inhibited by the shuttering events, and therefore the Schrödinger cat lives longer. This is a manifestation of the quantum Zeno effect. On the other hand, for  $r = 0.1$ , i.e., in the case of “out of resonance” reservoir, the peak of the Wigner function decays faster, indicating a rapid passage from a quantum superposition to a statistical mixture of the coherent states  $|\alpha\rangle$  and  $|- \alpha\rangle$ . This is a manifestation of the anti-Zeno effect. Summarizing, by manipulating the parameters of the engineered reservoir one can control the border between the quantum and the classical worlds.

### B. Zeno and anti-Zeno dynamics of Schrödinger cat states: the number probability distribution

We now consider the time evolution of the number probability distribution. The dynamics of this quantity in presence of an artificial shuttered reservoir coincides, as we have noticed, with the dynamics of a system interacting with a natural environment in presence of non-selective energy measurements. Therefore the results here illustrated apply to both scenarios. Moreover, as it is shown in Refs. [16, 18], the expressions for those observables which are diagonal in the Fock state basis does not depend on the secular approximation performed to derive the master equation (1), therefore the results

presented in this section go beyond the secular approximation too.

Following the lines of the Markovian derivation we solve directly Eq. (10) and obtain

$$\begin{aligned}
 P_n(t) = & \frac{2\mathcal{N}e^{-\alpha^2}}{a_t + 1} \sum_{j=0}^l \frac{l!}{j![(l-j)!]^2} \left[ \frac{a_t}{a_t + 1} \right]^j \\
 & \times \left[ \frac{\alpha^2 e^{-b_\tau t}}{(a_t + 1)^2} \right]^{l-j} \left\{ \exp \left[ \left( 1 - \frac{e^{-b_\tau t}}{a_t + 1} \right) \alpha^2 \right] \right. \\
 & \left. + (-1)^{l-j} \exp \left[ - \left( 1 - \frac{e^{-b_\tau t}}{a_t + 1} \right) \alpha^2 \right] \right\}, \quad (34)
 \end{aligned}$$

with  $a_t$  and  $b_\tau$  given by Eqs. (27)-(28), respectively. The quantity  $a_t$  is the difference between the mean quantum number of the system oscillator at time  $t$ , whose dynamics is studied in Ref. [8], and the initial mean number of excitations.

We consider once more the case of a high  $T$  reservoir with Ohmic spectral density, as given by Eq. (31). In Fig. 3 we plot the behavior of the number probability distribution. These plots confirm the results of the previous section. Also by looking at the number probability distribution, indeed, one sees that the quantumness of the initial state, indicated by the absence of the odd number components, can be prolonged or shortened in time, compared to the situation in which no shuttering/measurements are present.

## V. CONCLUSIONS

In this paper we have investigated the dynamics of a damped harmonic oscillator subjected to a series of non-selective energy measurements performed at time intervals  $\tau$ . We have assumed that the system is prepared initially in a Schrödinger cat state, i.e., a quantum superposition of two distinguishable coherent states. This state is strongly sensitive to decoherence induced by the external environment. The larger is the “separation” between the two components of the superposition, the faster is the decay into a statistical mixture of the two components.

We have shown, however, that when  $\tau$  is smaller or of the same order of the reservoir correlation time  $\tau_R$ , then the passage from a quantum superposition to a classical statistical mixture may be controlled. In more detail one can inhibit or enhance the “life” of the Schrödinger cat depending on some system/reservoir parameters. This result, which is based on the occurrence

of the quantum Zeno or anti-Zeno effects, respectively, opens new possibility for protecting very fragile states like the Schrödinger cat states from the destructive effects of the external environment.

We have also proved that one can mimic the effect of nonselective measurements by modulating the system-reservoir coupling constant. In this case one can use appropriately engineered reservoirs to implement experiments aimed at moving in a controlled way the quantum-classical border via the Zeno and anti-Zeno effect.

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